

homework 6

cs201.1

due 17 march 1999

real problems

1. Prove that the Cartesian product operator distributes over intersection. (Translated into symbols, prove:

$$X \times (Y \cap Z) = (X \times Y) \cap (X \times Z) \text{ .)}$$

2. Let f be a function which is one-to-one. Let g be a function which is not one-to-one. For each of the following functions determine if the function is 1) one-to-one, 2) not one-to-one, or 3) cannot be determined with the information given. Give reasons for your answer

a) $f \circ f$

b) $g \circ g$

c) $f \circ g$

d) $g \circ f$

3. Let R be a relation which is reflexive and transitive. Prove that $R^n = R$ for all positive integers n . (Hint: Show that $R \circ R = R$ and then use mathematical induction.)

4. Let R be a relation which is symmetric. Prove that R^n is symmetric for all positive integers n .